A Quest for Knowledge

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Motivation

In his 1945 letter to Roosevelt—*Science, the Endless Frontier*—which paved the way for the creation of the NSF, Vannevar Bush emphasizes the value of research for society and the importance of scientific freedom.

But...

- How do researchers act under scientific freedom?
- What are implications for the evolution of knowledge?
- How can funding institutions affect the researchers' actions?

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We apply the model to classical questions in the economics of science.

- How can society improve the evolution of knowledge?
- What is the optimal funding mix to improve researchers' question selection?

Literature

- **Economics of science:**
  Aghion, Dewatripont and Stein (2008), Bramoullé and Saint-Paul (2010), Foster, Rzhetsky and Evans (2015), Fortunato et al. (2018), Iaria, Schwarz and Waldinger (2018), Liang and Mu (2020), ...

- **Discovering a Brownian path:**
  Rob and Jovanovic (1990), Callander (2011), Garfagnini and Strulovici (2016), Callander and Clark (2017), Prendergast (2019), Bardhi (2020), Bardhi and Bobkova (2021), ...

Agenda

1. Model of Knowledge
2. Benefits of Discovery
3. Researcher's Choices
4. Applications: Moonshots & Research Funding

Model of Knowledge

Questions: Each $x \in \mathbb{R}$ is a question.

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Knowledge: Set of known question-answer pairs

$$\mathcal{F}_k = \{(x_1, y(x_1)), \ldots, (x_k, y(x_k))\}\text{, with } x_1 < x_2 < \ldots < x_k.$$

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Knowledge partitions questions into **research areas**

$$\{\underbrace{(-\infty, x_1)}_{\text{area } 0}, \underbrace{[x_1, x_2)}_{\text{area } 1}, \cdots, \underbrace{[x_{k-1}, x_k)}_{\text{area } k-1}, \underbrace{[x_k, \infty)}_{\text{area } k}\}.$$

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Research area i has **length** $X_i := x_{i+1} - x_i$.

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Brownian path determines answers:

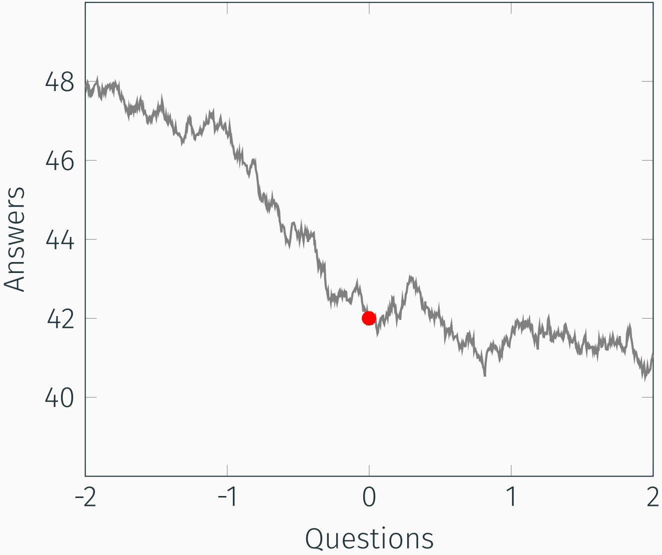
→ Conjectures are normal distributions.

→ Existing knowledge $\mathcal{F}_k$ determines mean and variance of conjectures.

$$Y(x|\mathcal{F}_k) \sim \mathcal{N}\left(\mu_x(Y|\mathcal{F}_k), \sigma_x^2(Y|\mathcal{F}_k)\right)$$

Model of Knowledge - Graphically

Truth and Knowledge



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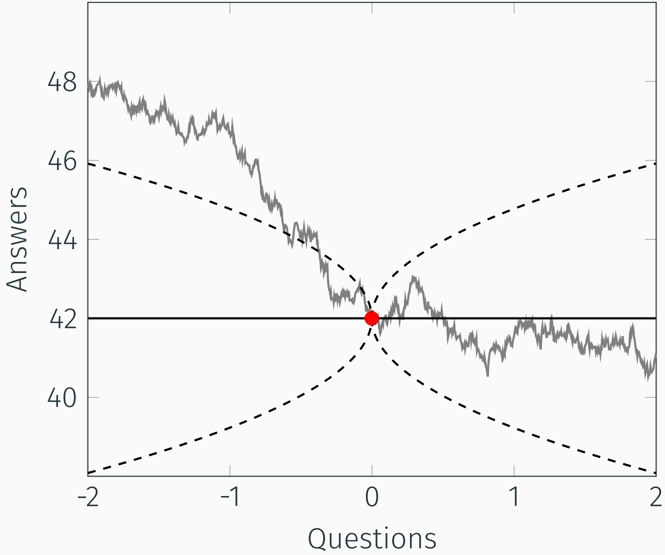
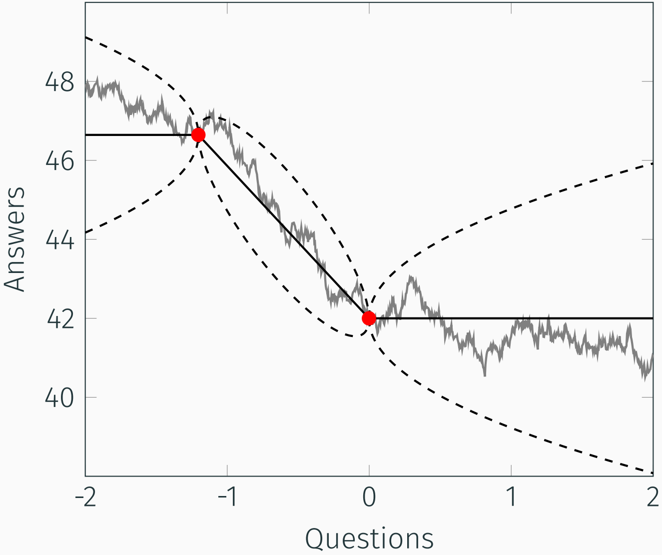


Expanding Knowledge



Benefits of Discovery

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with per-question payoffs

$$u(a(x), x) = \begin{cases} 0 & \text{, if } a(x) = \varnothing \\ 1 - \frac{(a(x) - y(x))^2}{q} & \text{, if } a(x) \in \mathbb{R}. \end{cases}$$

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Status quo guarantees a finite payoff but proactive choices can be beneficial if sufficiently good—error tolerance of $\sqrt{q}$.

The Value of Knowledge

Knowledge benefits society via more precise conjectures about optimal policies.

$$a^*(x|\mathcal{F}_k) = \begin{cases} \mu_x(Y|\mathcal{F}_k) & \text{, if } \sigma^2_x(Y|\mathcal{F}_k) \leqslant q \\ \varnothing & \text{, if } \sigma^2_x(Y|\mathcal{F}_k) > q \end{cases}$$

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Society's value of knowledge is

$$v(\mathcal{F}_k) := \int_{-\infty}^{\infty} \underbrace{\max\left\{1 - \frac{\sigma_x^2(Y|\mathcal{F}_k)}{q}, 0\right\}}_{=u(a^*(x),x)} dx.$$

Benefits of a Discovery

The discovery of an answer $y(x)$ to question x enhances knowledge

$$\mathcal{F}_{k+1} = \mathcal{F}_k \cup (x, y(x)).$$

The benefit of a discovery is the improvement in society's decision making

$$V(x; \mathcal{F}_k) := v(\mathcal{F}_k \cup (x, y(x))) - v(\mathcal{F}_k).$$

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x_1 and x_k are the frontiers of knowledge. A discovery

- expands knowledge if $x \notin [x_1, x_k]$ and
- deepens knowledge if $x \in [x_1, x_k]$.

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Benefits of discovery $V(d; X)$ determined by the question's distance to existing knowledge d and the length of the research area X. ▸ Characterization $V(d; X)$

Benefits-Maximizing Distance and Area

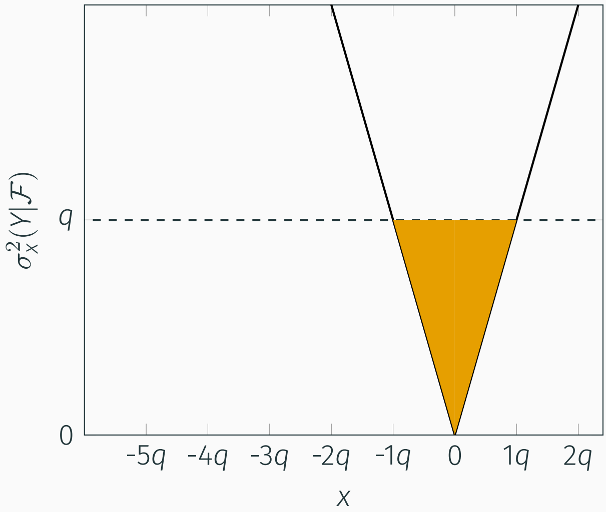
Proposition

The benefits-maximizing distance $d^0(X)$ in a research area of length X has the following properties:

- *When expanding knowledge, $d^0(\infty) = 3q$.*
- *When deepening knowledge in a*
  - *short area, $d^0(X) = X/2$,*
  - *long area, $d^0(X) \in (3q, X/2)$.*

There is a cutoff area length $\hat{X}^0 \in (4q, 6q)$ such that deepening knowledge dominates expanding knowledge iff there is an area with length $X \geqslant \hat{X}^0$.

Benefits of Expanding Knowledge



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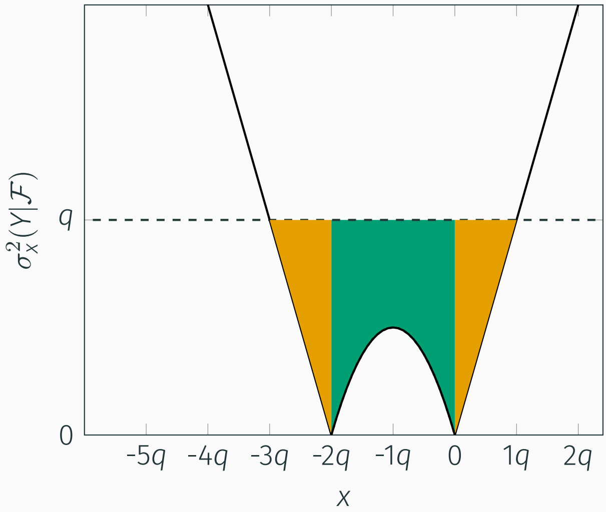

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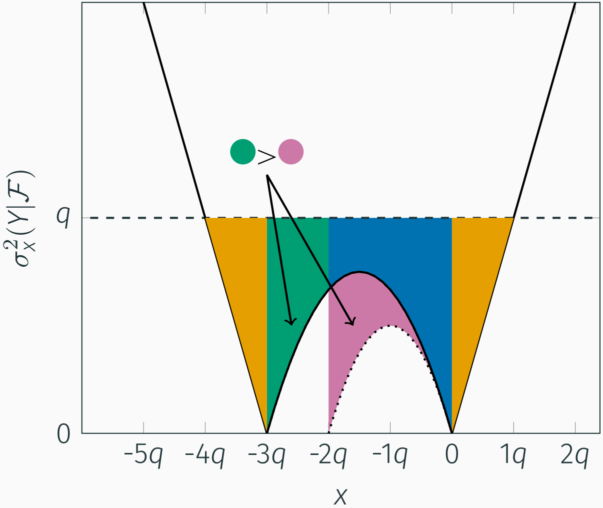

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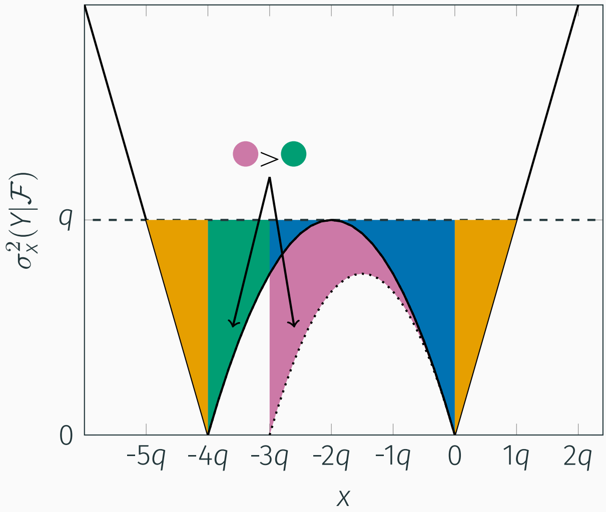

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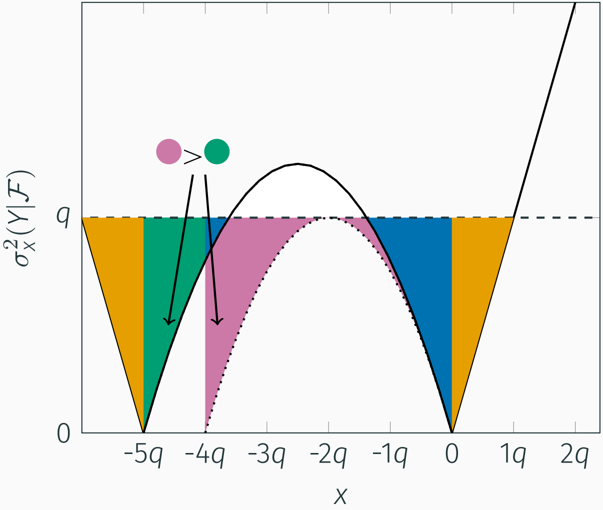
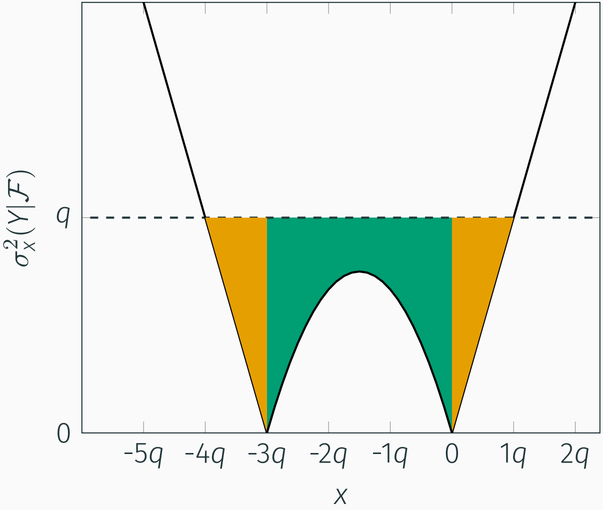


Benefits of Optimally Expanding Knowledge



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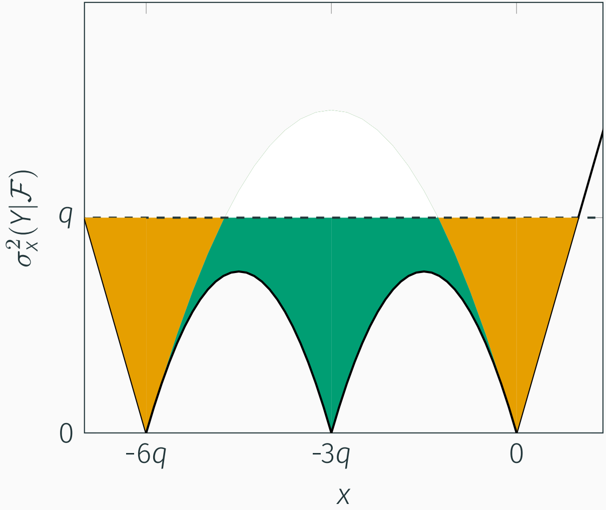

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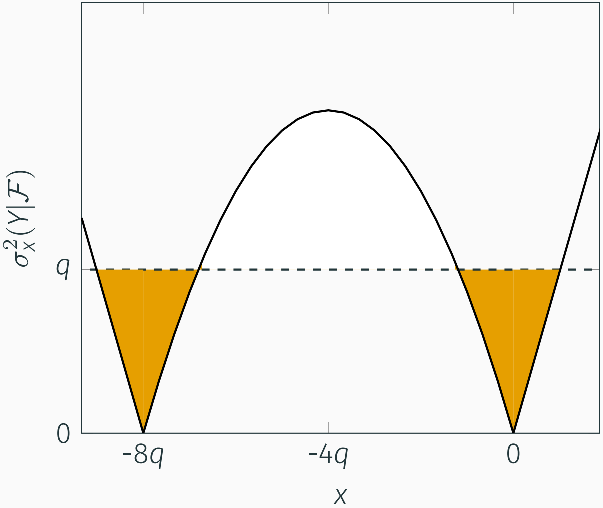

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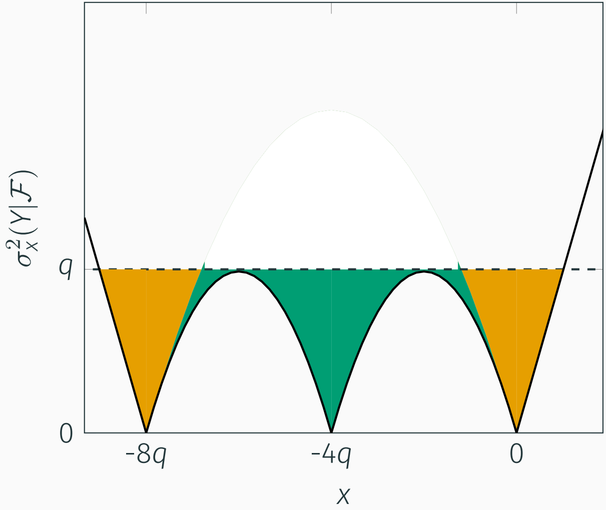

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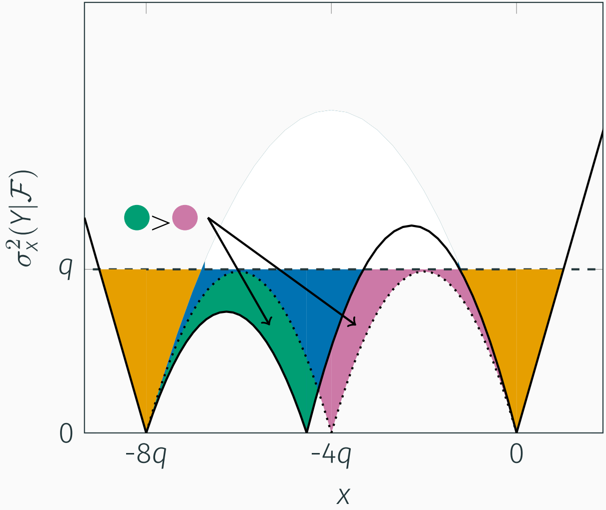

Researcher’s Choice

Researcher's Decision Problem

Researcher observes existing knowledge$\mathcal{F}_k$.

Researcher decides on a research question x in an area of length X with distance d and a success probability of research ρ.

Researcher's payoff consists of the benefit of discovery, $V(d, X)$, and a microfounded cost of research, $c(d, X, \rho) = \left(erf^{-1}(\rho)\right)^2 \sigma^2(d, X)$.

▸ Microfoundation $c(d, X, \rho)$

$$\max_{X \in \{X_0, \ldots, X_k\}} \underbrace{\max_{\substack{d \in [0, X/2], \\ \rho \in [0,1]}} \rho V(d; X) - \eta\, c(\rho, d; X)}_{=: U_R(X)}$$

Optimal Choice: Distance, Novelty and Research Area

Proposition

Suppose $\eta > 0$. There are area lengths $\widehat{X} \in (3q, 6q)$ and $\check{X} \in (6q, 8q)$ such that:

- *The researcher expands knowledge if and only if all available research areas are shorter than $\widehat{X}$.*
- *The researcher's payoffs, $U_R(X)$, are single peaked with a maximum at $\check{X}$.*

▸ Properties of optimal (d, X, ρ)

Application: Moonshots

Incentivizing Moonshots

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In period 1, society can costlessly pick x and ρ.

Optimality of Moonshots

Society maximizes

$$\max_{X,\rho} \mathbb{E}\left[\sum_{t=1}^{\infty} \delta^{t-1} v(\mathcal{F}_{t+1})\right].$$

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Proposition

There is a non-empty interval $(\underline{\eta}, \overline{\eta})$ such that the decision maker strictly prefers a moonshot in $t = 1$ for any $\eta \in (\underline{\eta}, \overline{\eta})$ provided δ is larger than a critical discount factor $\underline{\delta}(\eta) < 1$.

Moonshots and Evolution of Knowledge ($\eta = 1/8 \Rightarrow d^{\infty} = 2.1q$)

Evolution without Moonshot

Evolution with Moonshot

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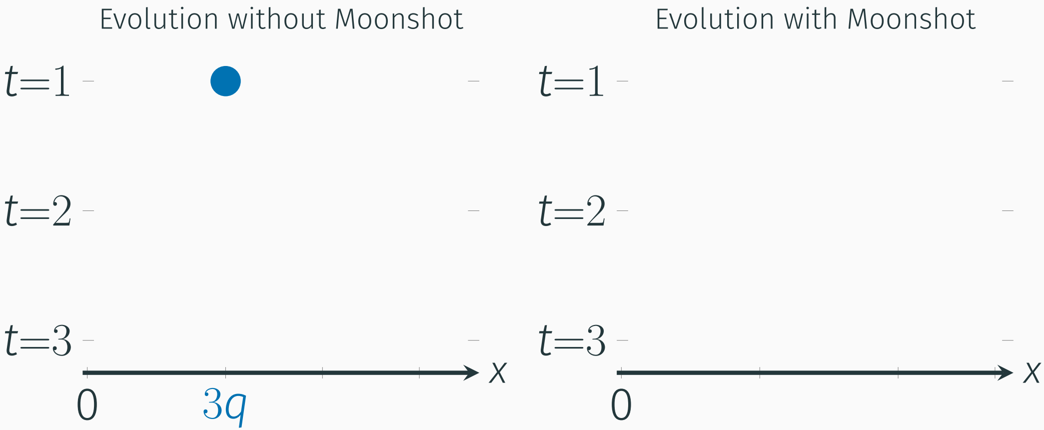

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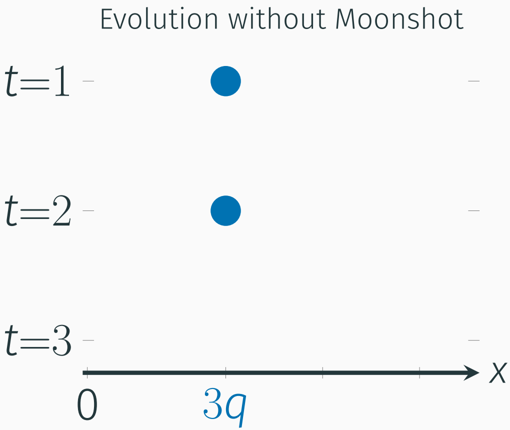

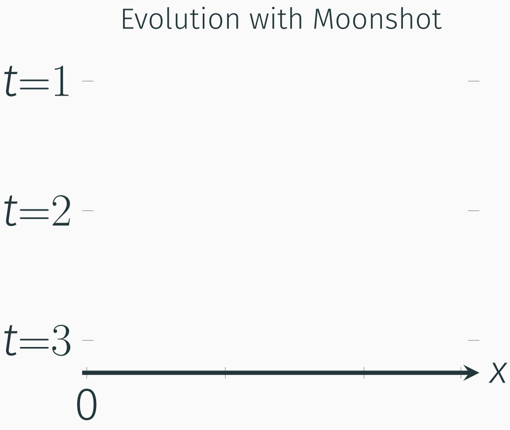

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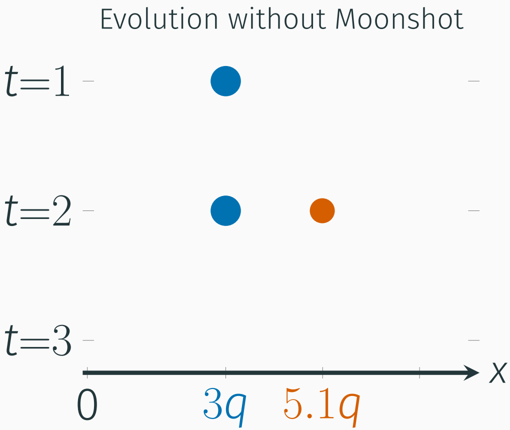

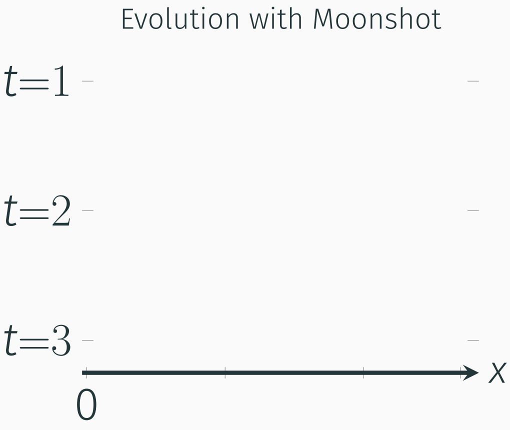

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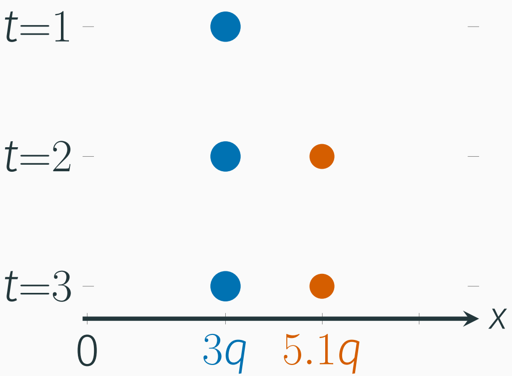

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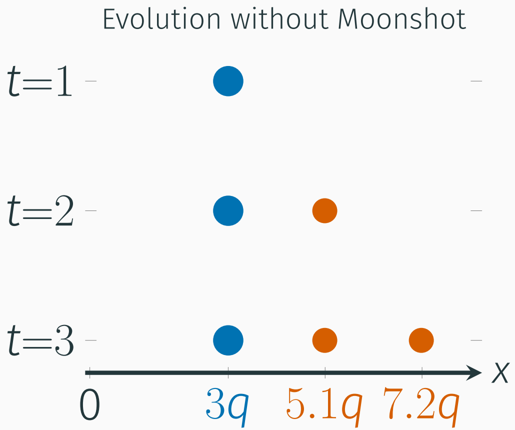

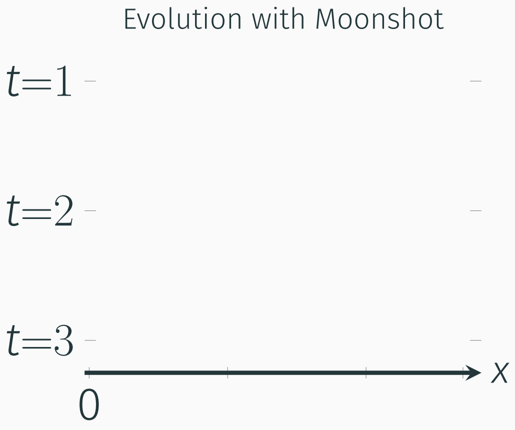

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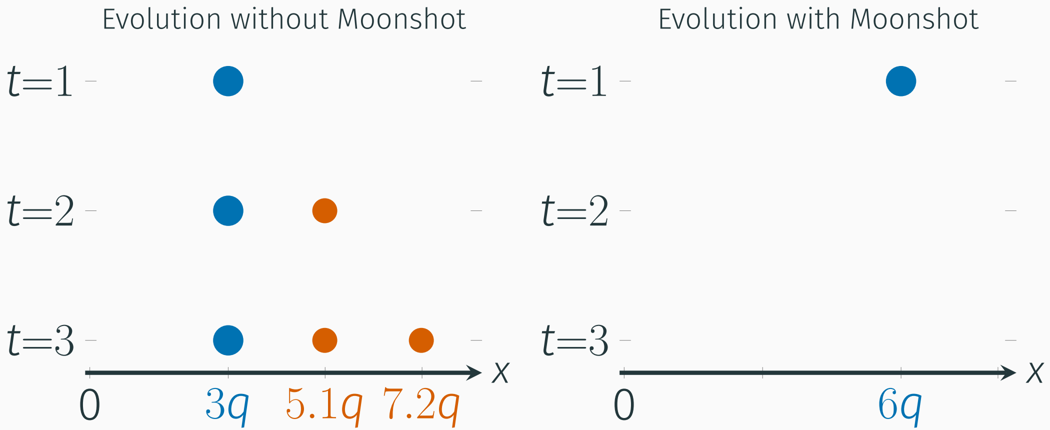

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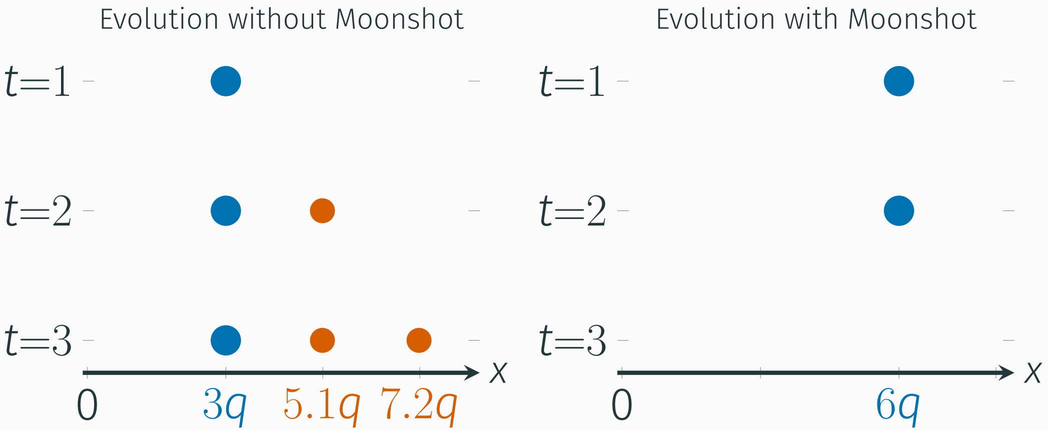

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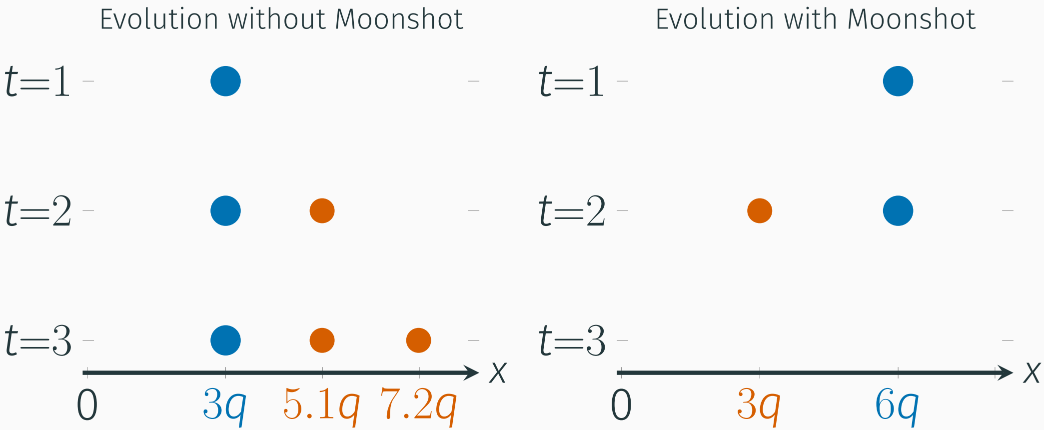

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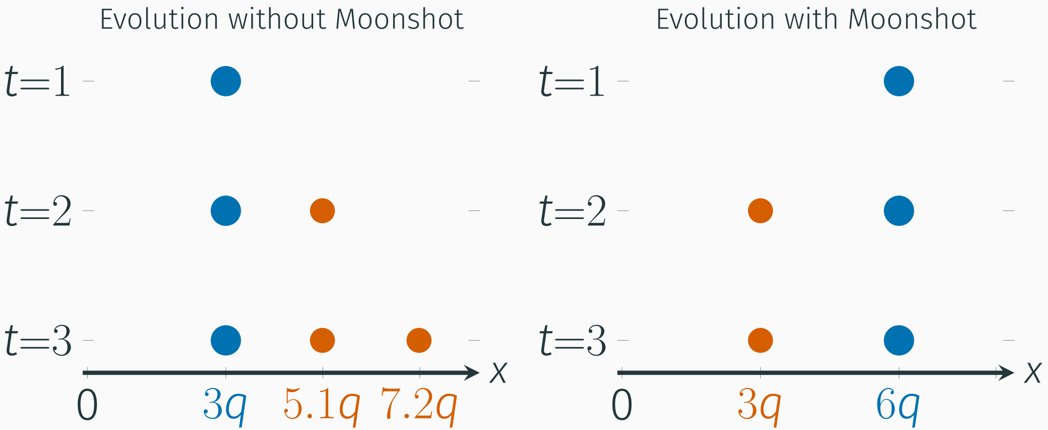

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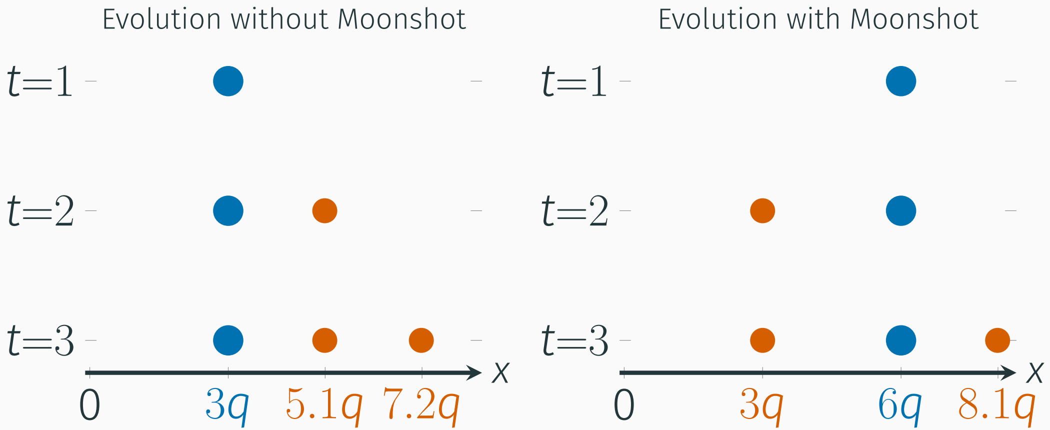

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Assume a funder with budget K has two instruments with relative price κ:

1. Cost reductions: lowering a researcher's cost by h, $\eta = \eta_0 - h$.
2. Prizes: awarding a prize ζ with probability $f(\sigma^2(d; \mathcal{F}_k))$.

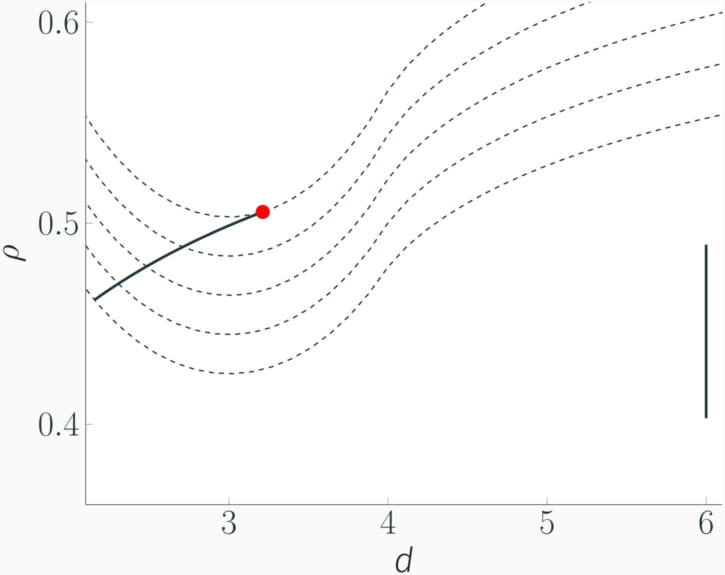
Proposition

A long-lived funder must use rewards to incentivize a moonshot with $d > 3q$.

A myopic funder might use rewards and incentivize excessive novelty $d > 3q$ as rewards introduce complementarities between novelty and output.

▸ Details

Optimal Funding - Short-Run Value of Research



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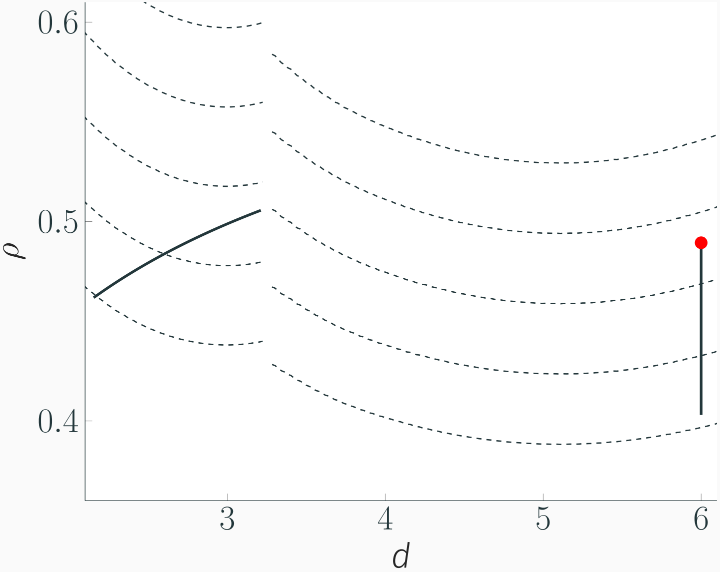

Conclusion

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We propose a model of knowledge built on

1. a large pool of questions,
2. knowledge informing conjectures about related questions,
3. society applying knowledge to choose policies.

We conceptualize research as the

1. free choice of research questions and
2. and the costly search for their answers.

Our model

- endogenously links novelty and research output and
- highlights the importance of existing knowledge for both (i) research and (ii) knowledge accumulation.

Benefits of Discovery - Characterization

Proposition

Consider a discovery $(x, y(x))$ in a research area of length X with distance to existing knowledge d. The benefit of the discovery is

$$V(d;X) = \frac{1}{6q}\Big(2X\sigma^2(d;X) + \mathbf{1}_{d>4q}\sqrt{d}(d-4q)^{3/2} + \mathbf{1}_{X-d>4q}\sqrt{X-d}\,(X-d-4q)^{3/2} - \mathbf{1}_{X>4q}\sqrt{X}(X-4q)^{3/2}\Big).$$

◂ benefits

Cost of Research

Research as Search for an Answer

The researcher searches for an answer $y(x)$ by sampling an interval $[a, b] \subseteq \mathbb{R}$.

The researcher discovers the answer $y(x)$ iff $y(x) \in [a, b]$.

Searching for an answer is costly: $c([a, b]) = (b - a)^2$.

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Lemma

Given a question x with distance d in a research area of length X, the lowest-cost search interval such that the answer is contained in the interval with probability ρ has cost

$$c(\rho, d; X) = 8\big(erf^{-1}(\rho)\big)^2 \sigma^2(d; X).$$

back

Output & Novelty: Substitutes or Complements?

Proposition

Suppose $\eta > 0$.

1. *When the researcher expands knowledge, distance, d, and probability of discovery, ρ, are substitutes.*
2. *When the researcher deepens knowledge, d and ρ are*
   - *independent if $X \leqslant 4q$,*
   - *complements if $X \in (4q, \frac{5}{2-\sqrt{3/2}}q)$,*
   - *substitutes for $d \in (0, \hat{d}(X))$ and complements for $d \in (\hat{d}(X), \frac{X}{2})$ if $X \in (\frac{5}{2-\sqrt{3/2}}q, 8q)$,*
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Why Substitutes and Complements?

A ceteris paribus increase in novelty affects both

- the marginal benefit of ρ, $V(d;X)$, and
- the marginal cost of ρ, $\frac{\mathrm{d}}{\mathrm{d}\rho}\left(erf^{-1}(\rho)\right)^2\sigma^2(d;X)$.

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$\frac{\sigma_d^2(d;X)}{\sigma^2(d;X)}$ is increasing and concave in X.

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For $X < 4q$, $V(d;X) \propto \sigma^2(d;X)$ implying that d and ρ are independent.

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When X just exceeds $4q$, the increase in $\frac{V_d(d;X)}{V(d;X)}$ accelerates as questions addressed proactively that were not before. d and ρ are complements.

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As X increases, $\frac{\sigma_d^2(d;X)}{\sigma^2(d;X)}$ dominates for small d where $\frac{\sigma_d^2(d;X)}{\sigma^2(d;X)}$ is highest implying that d and ρ are substitutes.

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As $d \to X/2$, the marginal cost effect $\sigma_d^2 \to 0$ implying that if $V_d(d;x) > 0$ d and ρ are complements.

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Whenever d is such that $V_d(d;X) < 0$, d and ρ are substitutes.

Optimal Choice: Distance, Novelty and Research Area

Proposition

Suppose $\eta > 0$. There is a set of cutoff values $\widehat{X} \leqslant \dot{X} \leqslant \check{X} \leqslant \widetilde{X} < 8q$ such that the following holds:

- *The researcher expands knowledge if and only if all available research areas are shorter than $\widehat{X}$.*
- *The researcher's payoffs, $U_R(X)$, are single peaked with a maximum at $\check{X}$.*
- *The optimal choices of distance, $d(X)$, and probability of discovery, $\rho(X)$, are non-monotone in X. The probability $\rho(X)$ has a maximum at $\dot{X}$, the distance $d(X)$ at $\widetilde{X}$.*

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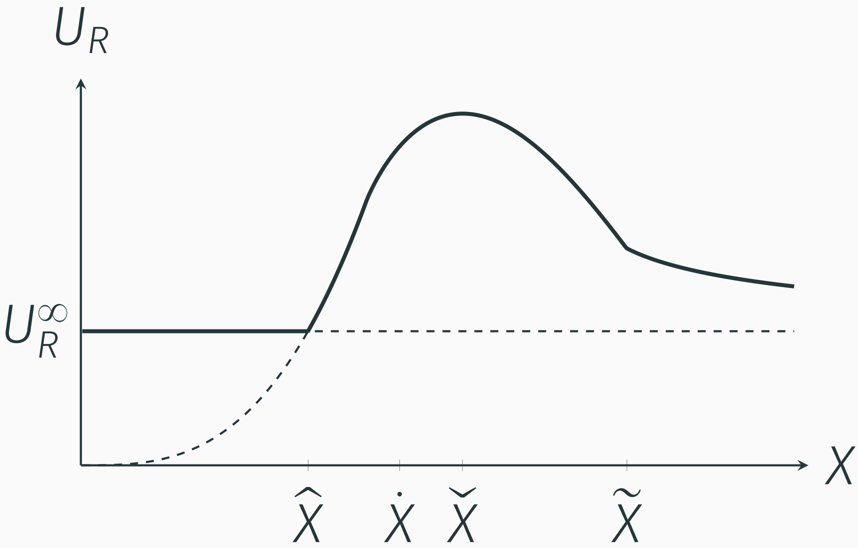
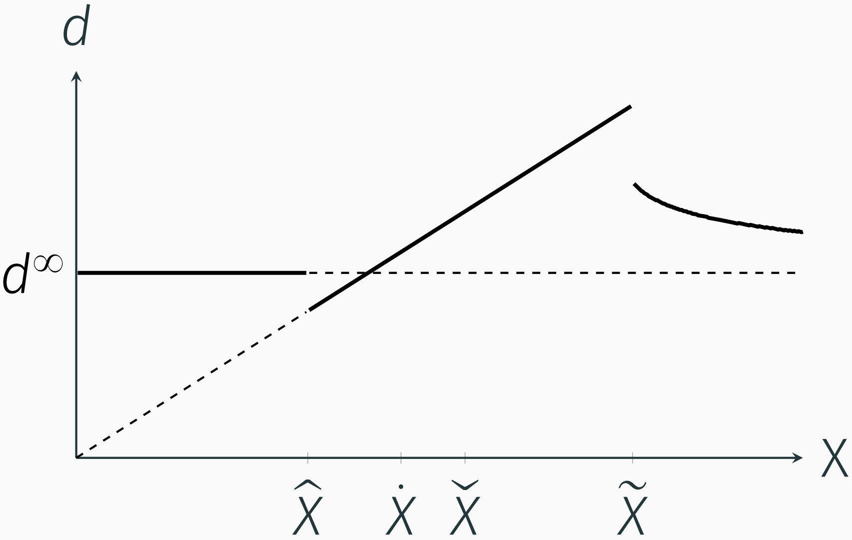
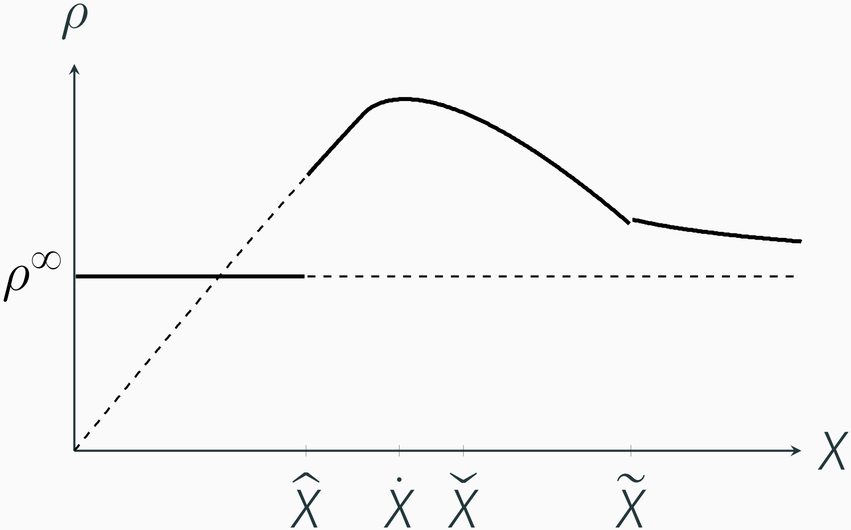
Researcher's Value by Area Length



Novelty by Area Length



Output by Area Length



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Proposition

The research-possibility frontier $d(\rho; \kappa, K)$ defined over $[\underline{\rho}(\kappa, K), \overline{\rho}(\kappa, K)]$

$$d(\rho; \kappa, K) = \min\{6q(K + s - \kappa\eta^0)\frac{\rho\tilde{c}_\rho(\rho) - \tilde{c}(\rho)}{2s\rho\tilde{c}_\rho(\rho) - s\tilde{c}(\rho) - \kappa\rho}, s\}.$$

To incentivize any $d > 3q$, the funder must award prizes for discoveries.

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